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Political Growth and Agricultural Intensification: An Extension of the Valley of Mexico Model

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In 1981 I proposed a mathematical model that allows one to derive archeological estimates of tribute flow between settlements, using scatter diagrams of site size versus catchment productivity. Arguing from exactly the same model, I will now outline a method that may be used (at least ideally) to monitor changes in the intensity of food production through time. This method, when applied to data on Formative period settlements in the Valley of Mexico, yields results that are consistent with accepted reconstructions of agricultural development in this area and therefore lends some support to my earlier interpretations.

To avoid needless repetition, the following discussion considers only a few basic aspects of the model that are directly relevant to the method being presented. The model's rationale, its limitations, and its previous applications have all been described at length elsewhere (Steponaitis 1981, 1983, 1984, 1985; cf. Finsten et al. 1983; Hirth 1984; Ackerly and Young 1984). Any reader not familiar with the model is urged to consult these references before proceeding further.

The Mathematical Basis

The model is designed to describe the settlement systems found in politically centralized societies that lack market economies. The idealized situation envisioned is one in which commoner households not only produce their own food, but also allocate a certain fraction of their yearly production as tribute that supports the political establishment. This establishment may consist of chiefs, high-ranking individuals, various political and religious functionaries, subsidized craft specialists, and other nonproducers attached to chiefly households. A regional hierarchy of settlements can thus be defined: subordinate settlements, called *villages,* pay tribute to local and regional *centers,* where much of the political establishment resides (Steponaitis 1981:322–325).

The model rests on the fundamental premise that a settlement's size is directly proportional to its food supply. In societies of the sort just described, the amount of food to which a settlement has access may depend on two things: (1) the productivity of the settlement's catchment (i.e., the annual yield of the productive zone directly exploited by the settlement's inhabitants), and (2) the flow of food as tribute either into or out of the settlement. Thus, for villages, the relationship between settlement size and catchment productivity is given by the following equation:

$$(1) \qquad V_{ij} = k\,(1 - t_1)\,P_{ij}$$

where V_{ij} is the population of the jth village in the district of the ith center, P_{ij} is the productivity or annual yield of that village's catchment, t_1 is the proportion of the total yield that is paid to the center as tribute (i.e., the tribute rate), and k expresses the number of people that can be supported per unit productivity (Steponaitis 1981:325–330). Note that this equation is intended as a purely theoretical construct, in which V_{ij} is expressed in units of *people,* and P_{ij} is expressed in units of *absolute yield,* such as tons of grain, kilocalories, etc. Since neither of these variables is directly measurable in the Valley of Mexico case, we are forced to make two bridging assumptions in order to operationalize the model archeologically.

The first is that "the residential density is the same for all sites of a given period" (Steponaitis 1981:334). In other words, the population of a given village (V_{ij}) can be expressed as the residential area in hectares (A_{ij}) multiplied by a constant *(D)* expressing the average number of people per occupied hectare, or

$$(2) \quad V_{ij} = DA_{ij}.$$

Similarly, let us also assume that the annual yield (P_{ij}) is directly proportional to the number of hectares of arable land (H_{ij}) within a fixed radius of the settlement (Steponaitis 1981:334), or

$$(3) \quad P_{ij} = QH_{ij}$$

where Q is a constant expressing the ratio between the annual yield of the *actual* catchment (whatever its true size and shape may have been prehistorically) and the number of hectares of arable land within the *analytical* catchment (however defined by the archeologist).

Based on this information, we can now define more precisely the pattern that appeared on the scatter diagrams used in my analysis (Steponaitis 1981:Figures 5, 8, 11). Substituting Equations 2 and 3 into Equation 1, we obtain

$$(4) \quad DA_{ij} = k(1 - t_1)QH_{ij}$$

$$(5) \quad A_{ij} = \frac{kQ}{D}(1 - t_1)H_{ij}$$

Hence, when site size in hectares (A_{ij}) is plotted against my index of catchment productivity (H_{ij}), the villages should appear along a line whose slope (M_V) is defined as follows:

$$(6) \quad M_V = \frac{kQ}{D}(1 - t_1).$$

From this we can see that the observed slope of the line of villages depends on a number of factors, of which the tribute rate (t_1) is only one, and not necessarily the most important (cf. Ackerly and Young 1984:977, 980–981). Although it is probably reasonable to assume that k remained constant over the periods considered in the original analysis (i.e., it is unlikely that the basic nutritional needs of people changed very much), the same need not be true of D and Q. Indeed, there is considerable evidence that the intensity of agricultural production substantially increased in the study area from Middle to Terminal Formative times (Sanders 1976). Such intensification would clearly inflate the value of Q (roughly speaking, the yield per unit of measured catchment area), which would tend to counteract any decreases in slope due to escalating tribute rates.

In fact, to the extent that D (the residential density) and t_1 (the tribute rate) can be reconstructed archeologically, observed changes in slope can be used to track changes in agricultural intensity. Simply rearranging the terms of Equation 6, and assuming that k remains stable through time, we find that Q is directly proportional to the observed slope times the residential density, divided by one minus the tribute rate, or

$$(7) \quad Q \sim \frac{M_V D}{(1 - t_1)}.$$

If the premises of the model and my bridging assumptions hold true, the right side of this expression becomes a *relative* index of productive intensity, values of which can be legitimately compared between periods.

The Empirical Trial

Applying such an index to Formative period sites in the Valley of Mexico is problematical at best, since neither D nor t_1 have been estimated with a great deal of precision, and the observed slopes are based on relatively few points. Even so, it is instructive to make the attempt and to see what happens. Archeologists who work in the area have often assumed that Formative villages had residential densities on the order of 20 to 50 persons per hectare (Blanton 1972; Parsons 1971; Parsons, Kintigh, and Gregg 1983; Sanders, Parsons, and Santley 1979:34–40). Let us therefore take the midpoint of this range, or 35 persons per hectare, as the best available approximation for D, and presume (for lack of a reasonable alternative) that it applies to all three periods being considered. Using this value in Equation 7 together with the tribute rates and slopes that have been estimated for each period separately (Steponaitis 1981), we find that the intensity index increases gradually through time, from 1.62 in the Middle Formative to 1.99 in the Terminal Formative (Table 1). This result, although tentative, is consistent with the idea that farming systems intensified during the Formative and shows that my original model does not necessarily conflict with prevailing opinions on the agricultural history of the Valley of Mexico, as some have implied (Ackerly and Young 1984:984).

Clearly, better data on residential densities and tribute rates (as well as prehistoric field and irrigation systems) are necessary before this aspect of the model can be fully evaluated. In the absence of such data, the measure of agricultural intensity used here can at best be taken as a crude index, and its values should

Table 1
Estimates of the relative intensity of production for the Middle, Late, and Terminal Formative periods.

Period	Observed slope[a] (M_V)	Estimated tribute rate[b] (t_1)	Estimated intensity of production[c]
Middle Formative	.0463	0	1.62
Late Formative	.0438	.157	1.82
Terminal Formative	.0477	.162	1.99

[a]Nucleated villages only. In order to be consistent with the method used in estimating t_1, each slope is based on a regression line calculated to pass through the origin (Steponaitis 1981:346, Tables 6 [note d] and 9 [note d]).

[b]The estimates are taken from Steponaitis (1981: Tables 7 and 10).

[c]Calculated using Equation 7. A value of 35 persons per hectare is assumed for D.

be regarded with a great deal of caution. But it does point to a new way in which, at least theoretically if not always practically, an important economic variable can be monitored archeologically.

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